

## Block Diagrams and Projectivity

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In the articles [6], [13], [14] of the author it was suggested that a nucleons quarks are not only confined by the gluon exchange, but that they form together with the Gleason frame of a rgb-graviton a geometrical tetrahedron. A mathematical foundation for this is proved in [10] as a first theorem of a Boolean block structure for a Hilbert space subspace geometry. It is concerned with 3-cycles. The second theorem extends this to 4-cycles and shows why the 4-dimensional spacetime in physics needs the octonian space extension for the quantum range.

### I. HILBERT SPACE IN FINITE DIMENSIONS

Finite dimensional real, complex or quaternionic Hilbert spaces  $H_n$  have orthogonal decompositions. If  $U$  is a closed subspace, any vector  $x$  in  $H_n$  can be written as  $x = y+z$  with  $y \in U$  and  $z \in U^\perp$ . The projection operators  $P_U: H_n \rightarrow U$  split  $H_n$  into a subspace  $U$  and its orthogonal subspace as  $H_n = U + U^\perp$ . The algebraic model OML  $L = L(H)$  has these subspaces as elements provided with the orthogonality  $U \perp U^\perp = U$ . A dimension function exists on the orthomodular lattice OML.

Noncommuting operators  $P, Q$  generate noncommuting subspaces  $U_P, U_Q$ . The lattice  $L$  with the former orthogonality is the set theoretical union of its maximal Boolean subalgebras, called blocks. The associated operators of a block are commuting. There is a dimension function on  $L$ . For dimension 4 or less some results are used for a modern presentation of quantum structures.

Important is for  $H_n$  its projective geometrical use. For lines in  $H_n$  the projective axiom is that no parallels exist. Each pair of lines  $l_1, l_2$  have a common point  $p_{12} = l_1 \cap l_2$ . For a subspace lattice  $L$  it means that the modular law holds. In a Hasse diagram for  $L$  no pentagons are allowed in form of the order relation which for subspaces means set inclusion. The elements  $a, b, c, d, e$  form a pentagon in case  $a < b < c < d, a < e < d$  and no other elements are involved. The chain length fits not to a dimension function on  $L$ . Projectivity means for the real case that as space  $R^{(n+1)}$  is taken and the projective points in a real  $P^n$  are taken the lines through the origin of  $R^{(n+1)}$ . The

subspaces and vectors of  $P^n$  can be described by  $k$ -tuples  $v_j, j = 1, \dots, k$  of vectors  $v = [x_1, \dots, x_n, w] \in P^n$  where  $w$  is added for the norming of the  $R^{(n+1)}$  lines through its origin. For  $w = 1$  the affine space vectors are written like  $R^n$  vectors  $x = (x_1, \dots, x_n)$  and can have a bilinear form for a Hilbert space metric  $\langle x, x \rangle = \sum x_j^2$ . Orthogonal  $u, v$  vectors satisfy  $\langle u, v \rangle = 0$ . The former splitting of  $R^n$  as  $H_n$  with this orthogonality, extended to subspaces, applies for an associated subspace lattice  $L$ . On a 2-dimensional space  $U$  for

$L$  one can choose an origin  $B$  and two orthogonal base vectors at  $B$  for coordinates of  $U$  as a block  $\{0, u, u', 1\} \cong 2^2$ . Choosing  $n$  such sublattices  $\{0, u_j, u'_j, 1\}$  blocks  $U_j$  in a higher dimensional space  $H_k$  for the union of the blocks  $0$  (1) is the smallest (largest) lattice element, identifying all smallest (largest) elements of the  $U_j$ , the two other, orthogonal elements with  $\langle u_j, u'_j \rangle = 0$ , called atoms, have 1 as lattice join  $u_j \vee u'_j = 1$  and as meet  $u_j \wedge u'_j = 0$ . The ordered chains are  $0 < u < 1, u = u_j$  or  $u'_j$  form a lattice  $MON$ .

As an interpretation for systems of spins, for instance in an atomic kernel, the base vector  $u$  as  $(1 \ 0)$  is kept fixed and the vector  $u'$  is turned by a rotation to  $(-1 \ 0)$ . This can be Boolean abbreviated for spin up as 1 instead of  $(1 \ 0)$  and spin down as 0 for  $(-1 \ 0)$ . Then the  $MON_n$  members are taken for selecting in every block a spin up 1 (as  $u$ ) or spin down 0 (as  $u'$ ) member and changes of states for the  $n$  spins mean that some  $u$  are replaced by  $u'$ . Projectivity for changing spins up/down directions can be added through choosing a projective extended plane  $E = [y, z, w]$  in  $H_k$  where the spin vector is a normal to the plane. The state  $u \rightarrow u'$  change of the spin normal direction is done by a 360 degree rotation of the vector on a Moebius strip in  $E$ .

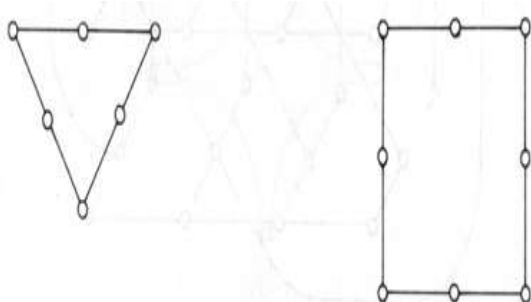
For higher dimensional blocks of an OML it is suitable to choose orthogonal bases  $(e_1, \dots, e_k)$  of  $H_k$ .

For  $k = 3$  and three local orthogonal  $xyz$ -space coordinates of spin, the projection operators have in a block of  $L$  three atoms as lines of the base vectors above the smallest element 0, three planes above the join of two atoms and their join 1 for the whole  $H_3$  space. Blocks having two atoms in common are identical since the complement of

their join is the third atom of the block. The blocks can have one atom in common. This means for different local bases  $(u_1, u_2, u_3)$ ,  $(v_1, v_2, v_3)$  that they can combine like two spins as  $s = (s_x, s_y, s_z) = (u_1, u_2, u_3)$  where one pair  $u_3, v_2$  are like spin up/down on one line parallel or antiparallel. If the first base is for spin and the second base for the electrical triple charge as  $v_1 = e$ , magnetic momentum as  $v_2 = \mu$  on one line with spin on  $u_3 = s$  and

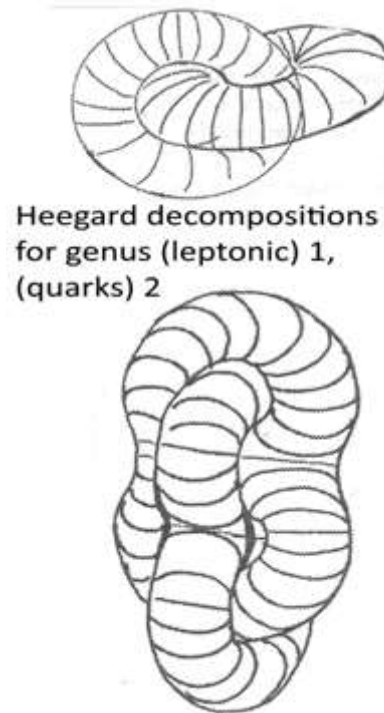
$v_3$  as the magnetic induction cross product of  $e, \mu$  then in the gyromagnetic relation the change of the sign means that  $+\mu$  (or  $-\mu$ ) comes from an orientation towards  $e$ . The Hopf map allows that  $e$  is on a latitude circle of a 2-dimensional unit sphere  $S^2$  of  $R^3$  which rotates for the  $e$  frequency counterclockwise  $+e$  or clockwise  $-e$ .

The lattice  $L$  of all blocks for  $H_3$  can be shown by drawing for every block an interval with two atoms at the end and the third in the middle of the interval. A theorem is for the subhypergraphs that no 3-cycles and no 4-cycles can occur. They violate the lattice structure of  $L$ .



**Figure 1** 3- and 4-cycles are not in the  $L$  of  $H_3$

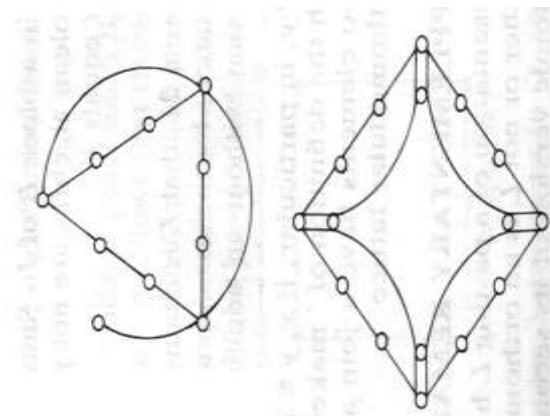
As an example, spin can also have for neutral leptons  $(w_1, w_2, w_3)$  helicity the  $u_1, w_2$  (momentum) vectors aligned. But then the two triples  $(v_1, v_2, v_3)$   $(w_1, w_2, w_3)$  cannot have two of their vectors aligned. A decay pair of an electrical charged lepton and partner neutral lepton occurs when a weak boson  $W^+$  or  $W^-$  generates them. As toroidal subgeometries of the weak interactions Hopf unit sphere  $S^3$  it is possible that the spins of the two leptons are identical in the former aligned description for their common spin (figure 2).



**Heegard decompositions for genus (leptonic) 1, (quarks) 2**

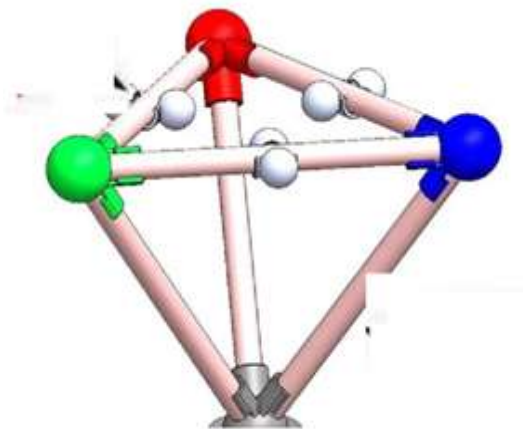
**Figure 2**  $S^3$  can have decompositions into two brezels, for leptons tori of genus 1, for quarks tori of genus 2 (lower figure);  $S^3$  is the geometry of the weak interaction  $SU(2)$  symmetry

For the case  $n = 4$  as spacetime coordinates the block structure with four points on an interval for the atoms is different. As well 3- as 4-cycles are allowed when additional block are inserted. The 3-cycles block, drawn in figure 3 curved is interpreted as an rgb-graviton as superposition of three color charge whirls from quarks.



**Figure 3** 3-cycle and 4-cycle in  $H_4$

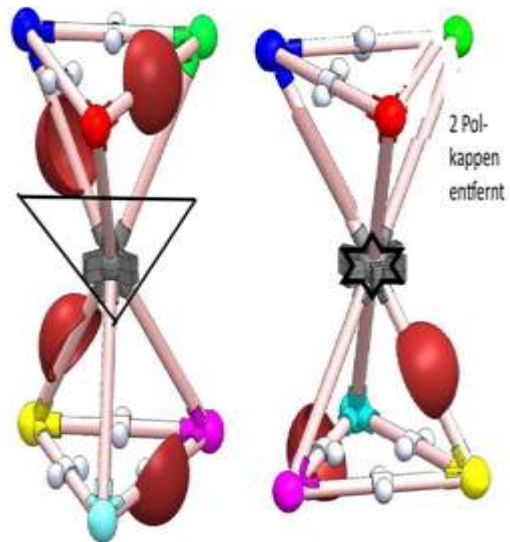
In nucleons they always show up having the three color charges red-green-blue. In the three blocks of the 3-cycle the 4 atoms are for two quarks as endpoints, presented by their color charge and the two points in the middle of their interval are for a gluon exchange between them in conjugate pairs. There are six gluons presented this way. The rgb-graviton has a time coordinate added to its three quark color charge atoms on the triangle. This is for its wave presentation which is experimentally verified. The proposed whirl presentation in nucleons is observed as neutral color charge of all nucleons. However, the interpretation as rgb-graviton is by now not accepted in physics. As a MINT-Wigris tool it is shown as an orthogonal base of the tetrahedron which has at its vectors ends the three quarks in form of a quark triangle. On the sides of the triangle are marked in figure 4 the gluons.



**Figure 4** three red, green, blue marked quarks on the upper triangle, gluons for the confinement of quarks marked on the triangle sides, an rgb-graviton whirl with conic tip at the lower tetrahedron vertex, spanning its 3 dimensions

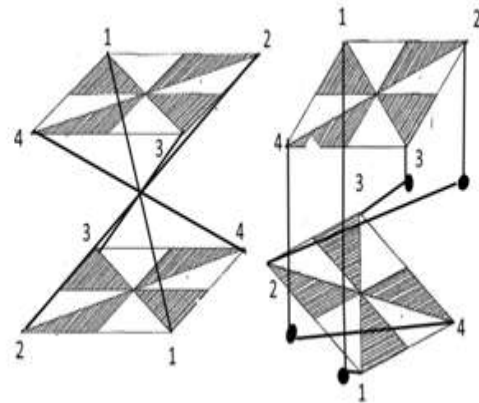
In fusion it is assumed that two protons are first in the position of the left figure with the triangles parallel in space and the rgb-graviton bases joined at the tetrahedrons tip. Since on one vertical interval between two quarks are then two u-quarks, there is a rotation in which one u-quark decays in the fusion by emitting a positron and a neutrino. After the rotation where the triangles form in the center as projection then a hexagon, on the three generated xyz-space coordinates are at the interval ends pairs of u-,d-quarks such that a 3- or 6-cycle in time can make an isospin exchange between them (figure 5). This way the proton and neutron exchange in every change of their states their

location between the upper and lower tetrahedron in a deuteron.



**Figure 5** fusion from two protons (at left) to deuteron (at right)

For 4-cycles in  $H_4$  it is assumed that they belong to two parallel condenser-like leptonic plates, using a figure for the magnetic group of order 8 (figure 6).



**Figure 6** octonian generation (at right) from xyzt-coordinates at left

The 4 atoms on an interval present four copies of the  $SU(2)$  generators, an identity and three Pauli matrices for spacetime coordinates (xyzt), drawn as diagonals between the two condenser plate quadrangles. In the 4-cycle, the spacetime coordinates are at the ends of the interval. In a rotation where the parallel quadrangles rotate such that in a central projection an 8-edge is obtained, the astroid in the center of the 4-cycle presents the octonian coordinates, doubling the

quaternions. The Heisenberg uncertainties arise as the 15 (position-momentum  $x,p$ ) and 46 (energy-time  $(t,f = E/h)$ ) atoms of the astroid, two new octonian coordinates 0,7 arise in 27 (angle  $\varphi$ ,  $\exp(i\varphi)$  function for waves) and 30 ( $z$ -coordinate and a spherical angle  $\theta$  of an octonian unit vector  $e_0$ , listed as 0, leaning towards the  $z$ -axis in this angle).

If the indices of the generated octonians are taken for listings of the seven octonian orthogonal spin-like Gleason frame GF triples then 123 is for space coordinates of spin, 145 is for the former mentioned electromagnetic  $(e,\mu,B)$  triple in superposition with spin, 347 is for rotations with an axis on 3 and a system rotating in the orthogonal 47 plane (neutrinos for instance), 246 is for heat with pressure on a volumes surface by its phonon actions, 257 can be used for Higgs bosons setting a mass scalar at a barycenter, 356 is for a nucleons strong interaction SI rotor as described in [1] and 167 is for the electromagnetic interaction.

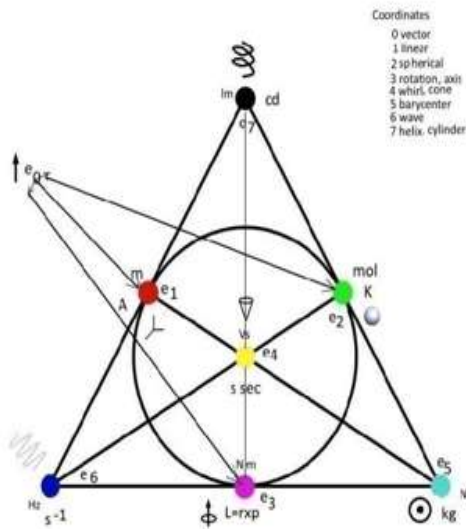


Figure 7 Fano memo for the seven octonian Gleason frame triples

Open for research is whether or not these base triples can have in  $H_4$  other quasiparticles where a GF quasiparticle like  $rgb$ -gravitons or spin can be GF replaced and can induce changes of composed systems states. In this case when in the part at left of figure 3 is used, the composed system has three subsystems as vertices of the triangle. Changing states of the composed system can occur on the triangles sides through other energy exchanging quasiparticles which are replacing gluons for the nucleon tetrahedron.

The right part of figure 3 has with central astroids another use. For the octonian 4-cycles astroid alternatives can be constructed. An astroid for  $xyzt$  vertices of a quadrangle can alternatively mean that the real coordinates are extended to complex numbers and  $(z_1, z_2, z_3, z_4)$  presents then a complex 4-dimensional space. According to experimental datas to be computed and of energies involved, a system can use one or another such coordinate extension. The  $xyzt$ -coordinates and energies presence are locally chosen in  $H_4$ , using projection operators, as well as the choice of GF triples and energies in the 3-cycle case.

## II. 2. PROJECTIVE FUNCTIONS, SHAPES AND SPACES

The projective geometry shapes arise often for systems in rotation. Before this is discussed the use of complex numbers of physics and of dihedrals  $D_3$  with the symmetry of complex cross ratios is investigated. After complex numbers are generated for their use in physics, for fusions SI rotor is observed that the  $rgb$ -graviton generated tetrahedron has the  $S_4$  symmetry of order 24. It is factorized by the CPT Klein group of order 4 to  $D_3$ , the symmetry of the quark triangle, which has as representation the SI rotor (figure 10 left). Permutations of its cross ratio numbers  $0,1,\infty$  are often used in physics, especially when computed or observed values tend to projective  $\infty$ . Complex numbers arise through the the conjugation operator  $C$  of physics. The Hopf map for  $SU(2)$  uses for its first coordinate the  $\sigma_1$  matrix which in multiplication sets the complex distance measure. This matrix reverses the sign of  $z = r \cdot \exp(i\varphi)$  to  $1/z = \exp(-i\varphi)/r$  for the complex conjugation.  $C$  reverses for a conic rotation the orientation between clockwise  $cw$  and counterclockwise  $mpo$ . In using a matrix extension it means that two base vectors in a  $(z,t)$ -plane are listed in the order  $(0 \ 1)$  first row or  $(1 \ 0)$  first row, When this order is taken for computing the orientation with the  $\sigma_1$  Pauli matrix, the vector is in the second row with the first row  $(1 \ i)$  and the determinant gives in the first case 1 and in the second case  $-i$ , which is the conjugation  $z = x + iy \rightarrow x - iy$ . For the electromagnetic interaction the change of sign means that its electrical or neutral charge is rotating  $mpo$  for  $+$  and  $cw$  for  $-$ . This is on the Hopf sphere  $S^2 = h(S^3)$  where the charge is a point on a rotating latitude circle with angular frequency  $\omega = 2\pi f$ . It sets for the space coordinate  $z$  and time  $t$  its complex use in  $z_1 = z + ict$ . The polar form for the  $z_1$  coordinate is obtained through the astroid where the octonian 32 coordinates are doubled and 3 is the radius vector on 0 while  $2 \ \varphi$  is the  $\exp(i\varphi)$



function on 7. The maps are  $z \rightarrow r$  and  $\phi \rightarrow \exp(i\phi)$ . The linear coordinates in the latitude plane get polar coordinates and the complex numbers are listed in  $2 \times 2$ -matrix form as  $x$ -id identity,  $y \cdot \sigma_2$  where the second Pauli matrix has as first row the base vector (0 1) and as second row (-1 0). In using here the other space coordinates in a plane orthogonal to the  $z$ -axis means that in the geometry of the Hopf sphere the  $z, t$  coordinates are the rotation axis though its north  $\infty$  and south poles. The stereographic map  $st$  makes a projection  $st \circ h(z_1, z_2 = x + iy)$  onto a tangent plane from  $\infty = (1 0)$  to the  $xy$ -tangent plane of  $S^2$  in the south pole of  $S^2$ . The complex coordinate in this plane is  $st \circ h(z_1, z_2) = z_2 / z_1$  for  $z_1 \neq 0$ .

The stereographic map is a central projection which belongs to gravity. Its general relativistic scaling factor of the Minkowski matrix is projective obtained as a Moebius transformation on  $S^2$ , using the Schwarzschild radius  $R_s$  of a central system  $Q$  like a sun together with its gravitational potential field which can let a second small system  $P$  rotate about  $Q$  when its momentum speed is between the first and second cosmic speed of  $Q$ . The projective coordinates  $[r, r', w]$  present a distance measure  $r = |QP|$  where  $Q$  measures its distance to  $P$ . For  $P$  a Gleason operator  $G'$  is applied to the usual  $\langle r, r' \rangle$  measure in  $\langle r G', r \rangle = \langle r - R_s, r \rangle$  and the projective  $w$ -coordinate set allows division to obtain  $[1, r' = (r - R_s) / r, 1]$  for the central projection as Moebius transformation  $MT$  with  $r = z$ . The system  $P$  measures its distance to  $Q$  unsymmetric as  $|PQ| = r - R_s$ . If  $R_s$  is normed to 1, the coefficient matrix  $G$  of the general relativistic scaling factor  $(r - R_s) / r = \cos^2 \beta$  has the first row (1 1) and the second row (1 0). It has the order 6. It is used for getting on a circle the 6th roots of unity as discrete rotations of an  $G$ -compass (figure 12) needle  $e_0$  which has for the 7th octonian coordinate a bounding projective circle  $P^1$  with coordinate  $\exp(i\phi)$ . The angle  $\beta$  is using an orthogonal spiralic gravitational projection, set for the metrical rescaling, between a line for the metrical differential  $dr$  and its projection upwards to a second line in an angle  $\beta$  as in figure 8 for the new differential

$dr' = dr / \cos \beta$ . For the metric, the area  $dr \cdot dt$  is preserved such that for the image  $dr' \cdot dt'$  the new time differential is  $dt' = dt \cdot \cos \beta$ .

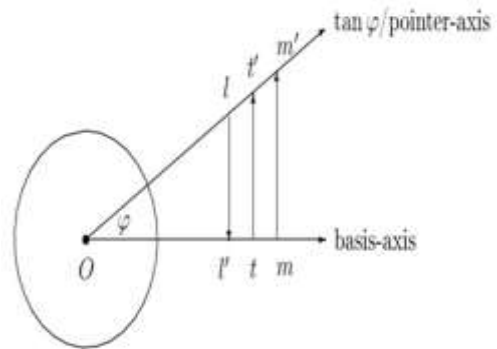


Figure 8 the angle  $\beta$  is in this figure replaced by an angle  $\phi$  between the two lines and  $l'$  as length has to be replaced by the differential  $dr$  for getting  $dr'$ , replacing  $l$  in this figure. Time  $t'$  is replaced by the differential  $dt$  and  $t$  by  $dt'$ ; in this figure also the Minkowski special relativistic rescaling is shown, setting  $\sin \phi = v/c$ ,  $v$  relativistic speed,  $c$  speed of light; the matrix transformation  $M$  of order 2 for this has the rows of  $G$  interchanged

The spiralic contraction/expansion action of gravity is observed when two galaxies  $P, Q$  form a common barycenter  $B$  and hit by their contracting spiralic rotation at  $B$ . In figure 8 the upper ray towards  $P$  at length  $l$  is contracted to the new distance  $|BP| = l'$  on the lower ray. For a length expansion, gravitons can add an additional accelerating speed with which  $P$  rotates about  $B$ . This is observed for a planet's orbit about a central sun as  $B$ . The projective computed Kepler ellipse for its orbit has for the larger ellipse diameter an additional turning angle such that with the increased speed the diagonal is turned by an added phase angle as in figure 10. It generates the rosette orbit of  $P$  about  $Q$  at  $B$ .

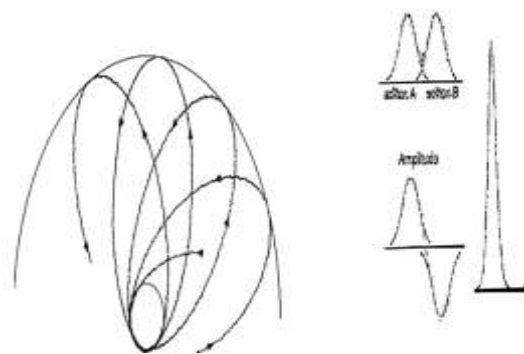


Figure 9 rosette orbit of a planet rotating about a central sun at left, soliton amplitudes which change density of matter at right

Gravitational waves have been observed for these actions of gravity. It is also available in the quantum range for a nucleon pulsation, postulated in the MINT-Wigris research project [1]. A video exists for this.

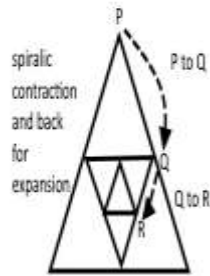


Figure 10 the rotation for the model SI rotor at left is by setting one axis and rotate the parts of the two other axes alternatively cw and mpo with a conic rotation, at right is added the gravitational spiralic contraction/expansion as a pendulum motion in time for a quark triangle of a nucleon

In the SI rotor model the rotation group is  $D_3$ , the dihedral group of order 6. It has one element of order 3 for a momentum vectors six rotation (part at right in figure 11) and it sets at every rotation one of the three barycentric coordinates (part at left in figure 11) of the nucleon triangle as axis for the conic rotations of the SI rotor. It is computed, using Gleason operators or states of an OML L, that the nucleons observed mass is additive coming from a states 10 percent of the quarks mass sum and about 90 percent come from inner speeds and interaction energies in the nucleon, using  $mc^2 = hf$ . The states or GF equation is available as a line in a projective octonian mass  $m$  5 and frequency  $f$  6 plane 56 with projective coordinates  $[m, f, w]$  as  $56w$ . It is observed that also the reverse transformation occurs where mass gets measured as frequency. In the matter wave computation it is necessary to rescale special relativistic mass with a speed  $v$ , having a frequency, which is used for the common momentum  $p = mv$  as group speed of the matter wave package. Some gravitational actions can also use solitons which change density as mass per volume (figure 9). The ripples with which they travel have an amplitude similar to those which are observed when graviton waves hit matter. Physics attributes to the Einstein spacetime ripples not a change of matter density, but observes the volumes length contraction/expansion.

If for electromagnetic waves EMI is assumed that its frequency is measured as mass when redshift or a broken direction of its world line are observed then the change of density through solitons means that energy frequency is emitted through solitons in the redshift case or when EMI waves hit matter and give up part of their energy. The broken world line means that its momentum direction gets a turning angle as in figure 8 such that the orthogonal projected mass/frequency rescaling can be applied. The double lensing is explained the same way: EMI waves absorb in this case mass energy from the star they are passing by and change their momentums direction such that doubled stars are observed instead of one which did send the EMI waves.

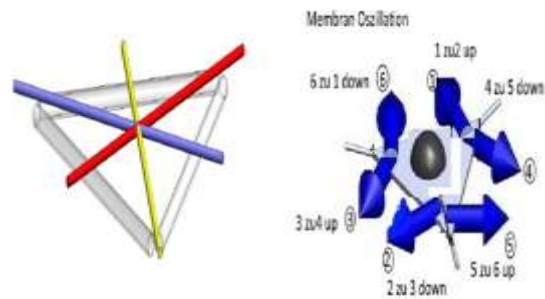


Figure 11 barycentric coordinates at left, at right six momentum states

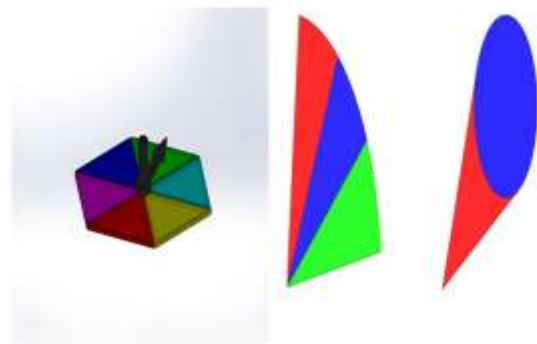


Figure 12 G-compass for the general relativity metrical rescaling factor at left, rgb-graviton (middle part), gluon cone at right, - the middle part has its red-green sides identified to form a conic whirl

The change of density as mass per volume has as threshold the Schwarzschild radius  $R_s$  or dark matter or black holes. In low temperature, gravity can collapse dimensions by projective duality.



**Figure 13** dark nucleon at left, dark matter/black hole (middle), dark whirl at right

The projective correlations can change a 3-dimensional system into a 1-dimensional. This happens in a 5-dimensional  $P^5$  which has in projective extended coordinates  $[x,y,z,ict,m,f]$ . Such a space is for field descriptions in octonian 123456 or complex  $(z_1, z_2, z_3)$  coordinates. The radius inversion for the density change is  $rr' = Rs^2$ , using the inversion at a circle of radius  $Rs$ . In figure 13 at left is a nucleon as a 3 lemniscates color charges butterfly of 3 quarks. They are joined at a singularity of a dark matter system (middle part of figure 13). This geometry is a Horn torus where for a torus the rotation axis has a singularity in common with the vertical transversal circles. In figure 13 at right is a figure for dark acoustic/heat whirls. The coil is actually missing and the two cones are joined at projective infinity by a circle for a pinched torus. A pinched torus arises also for EMI waves when their cyclinder is closed at projective infinity by a point, - the coil is then for the cylinder on which the EMI helix line is located. The mathematical inversion for EMI is for speed at the speed of light  $v'v = c^2$ . Inside a dark energy pinched torus for EMI energy speeds are higher than speed of light. The dark whirl inversion applies to this too. This inversion gives rise to a fourth Heisenberg uncertainty HU which Heisenberg missed:  $h$  for his lower HU bound is replaced by an upper  $c$  bound. The parts for the Kalmbach uncertainty are an octonian  $e_0$  turning angle  $\theta$  towards the space  $z$ -axis, used for  $n$ th roots of unity on the compass ( $n = 6$  in figure 12) [19]. The other part is coming from the octonian  $e_7$  EMI coordinate as a circle  $U(1)$  [15].

For other properties of gravity the authors bibliography can be consulted.

Quoted from another article of the author is a postulate for inertial mass:

Speed of the nucleon is kept constant and linear in space. In case a force is acting on its momentum, the mass is treated as inertial mass and differentiated as function to  $b/r^2$ ,  $b$  a known real

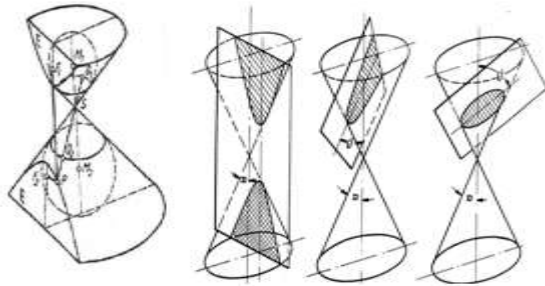
constant. Its direction as vector is set against the force such that energy is needed to move the nucleon away from its momentums world line. When integrated to a potential  $-a/r$  by the SI rotors action it sets a potential field POT outside the nucleon. It is inside the bubble a unified potential field for its quarks potentials. Quarks are energy systems for this, having a mass and an electrical charge (see also [11]). In the localized version, POT has two systems: EM(pot) is presented as nucleons positron for a proton and as a neutrino for a neutron. They are sitting on a Bohr shell above the nucleon as a kernel. E(pot) is the nucleon masses potential field, generated by the rgb-graviton in the nucleons outer environment.

Concerning projective shapes, the Kepler conic sections are mentioned. The orbits of planets P or comets, for which a common barycenter is set with a huge (central) sun Q are shown in figure 14.

The two cosmic speeds of Q are set by the gravitational potential and the Schwarzschild radius  $Rs$  of Q [14]. Three correlations in a projective extended space  $[x,y,w]$  are used. The  $xy$ -plane is generated by the radius  $r$  drawn from Q to P and the P momentums vector for its world line  $g$ . Then gravity changes the P orbits  $g$  shape to a conic section. Draw first a vertical axis B to the  $xy$ -plane through the center Q of a circle with diameter  $2r$ , containing P as point. Then B is shifted away from Q as center when the P momentums speed  $v$  is not the first cosmic speed  $v = v_1$ . For  $v_1 < v < v_2$  (second cosmic speed) the  $xy$ -plane gets a leaning angle  $\theta$  by applying  $e_0$  towards B. As the Kepler ellipse solution is computed, it uses the preservation of energy as  $E = \text{potential} + \text{kinetic}$  energy,  $E = E(\text{pot}) + E(\text{kin})$  and the preservation of rotational energy  $E(\text{rot})$  in its momentum form  $L = r \times p$ .

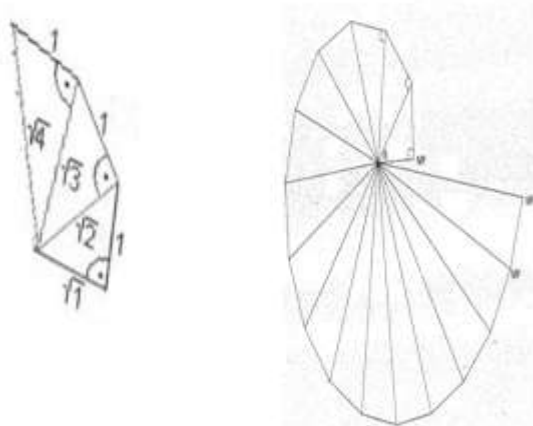
The projective quadric for  $w = 1$  is in the  $v = v_1$  case a circle  $x^2 + y^2 = r^2$ . For  $v_1 < v < v_2$  the equation is  $ax^2 + by^2 = 1$ . Beside a larger circle in the  $xy$ -plane for the large diameter of the ellipse for the largest position of Q to P there is a small concentric diameter for the nearest position of Q to P and Q stands in the nearest focus of the ellipse to this circle. The quadrics are for a correlation the points which are incident with their dual hyperplane which in 2 dimensions is a line. The free fall case  $v < v_1$  for the P speed is linear and for the escape cases  $v = v_2$  and  $v_2 < v$  the leaning angle  $\theta$  gets larger such that the plane intersets the double cone in a parabola where one conic line is parallel to the  $xy$ -plane and the intersection is a branch of a hyperbola for the P orbit when the leaning angle is more increased (figure 14).





**Figure 14** hyperbola at left, conic sections with parabola and ellipse added at left

The spiralic shapes when two galaxies form a common barycenter B and are spiralic attracted by gravity are drawn in form of orthogonal projections in figure 15 which for the rosette figure in 9 can also be a spiralic extension by adding to the Kepler computed location of P on a ray an additional orthogonal vector to a third ray where the added speed generates a larger (angle) distance between Q and P.



**Figure 15** spiralic roots at left, spiral at right with center B

Before the point B is reached, the two galaxies energies are transformed into the energy of a Higgs boson or dark matter Horn torus. Similar intermediate or permanent locations are obtained when two leptons or photons are hitting and form a weak boson as intermediate energy carrier. It decays into two particles again with the momentums of the input leptons exchanged for the two output particles. Physics draw for this the Feynman diagrams, deleting a description of the weak boson which is drawn as a point where the two momentums hit. The weak boson has a mass as the Hopf  $S^3$  unit sphere. Dark matter has in a Horn torus a huge mass compared to its small volume. The leptons mass is attached to these particles by a

kg GF with complex coordinates at the three base vectors added. There are also in the quark case for a kg GF six choices for setting the series masses. The series is then doubled to a 12 series by the conjugation operator C. Observable is the mass only by one of the GF base vectors for a member of the series.

### III. CONCLUSIONS

In this article all properties of gravity and its possible quasiparticles are presented: the first observations are for the projection operator calculus of Hilbert spaces in form of their subspace projected lattices. The 4-dimensional block structure shows in the theorems, proved in [10] that for a color charge block in figure 3, a rgb-graviton as whirl and superposition of three color charges exists. The part at left in figure 4 shows that a 1234 leptonic xyz-t-cycle must split the quadrangles coordinate vertices into two vectorial rays, named 15, 27, 30,46 for the indices of the eight octonian coordinates. The many shapes and used projective calculations for gravity are presented in section gravity uses central projections like the stereographic maps of unit spheres, it also uses orthogonal Hilbert space projections in many dimensions. The many applications which can be seen as interpretations or through exact computations include all observed properties of gravity.

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